**PROGRAMAÇÃO MINISSIMPÓSIO - CNMAC 2025**

# Teoria Espectral de Grafos

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## 1. Descriçao

Em Teoria Espectral de Grafos (TEG) associamos uma matriz $M$ a um grafo $G$ e estudamos como propriedades estruturais de grafos podem ser obtidas a partir das propriedades algébricas da matriz associada.

Em particular, dado que os autovalores e autovetores de $M$ estão diretamente relacionados com invariantes de um grafo (distância média, diâmetro, raio, número isoperimétrico, energia, para mencionar alguns), eles podem fornecer informações úteis acerca do grafo ou acerca de uma aplicação modelada pelo grafo. Por exemplo, em muitos modelos de comunicação, o diâmetro desempenha um papel chave no desenho de redes. A energia de um grafo (soma dos valores absolutos dos autovalores da matriz de adjacência) é estudada intensamente em Química e pode ser usada para aproximar a energia total dos $\pi$-elétrons de uma molécula. A Teoria Espectral de Grafos também aparece naturalmente em vários problemas da Física Teórica e da Mecânica Quântica. Além disso, TEG mostra crescentes conexões com outras áreas, em especial na área de ciência de dados por meio da programação semidefinida. Esta última iremos explorar em duas das palestras nesta proposta envolvendo pesquisadores da UFMG.

A TEG é uma área de pesquisa de muito prestígio internacional, com um crescente número de publicações em revistas reconhecidas. No Brasil, ela está em franco estágio de desenvolvimento com pesquisadores reconhecidos internacionalmente. A comunidade é muito ativa e mostra sua importância na área, especialmente em suas participações nas prestigiadas conferências internacionais do ILAS (International Linear Algebra Society).

## 2. Objetivos

Abaixo listamos os principais objetivos que pretendemos atingir com a realização da atividade:

- integração entre os vários grupos de pesquisa em TEG do Brasil;
- disseminar a pesquisa atualmente produzida pelos grupos de pesquisa de diferentes centros e dessa maneira promover uma interação entre seus membros;
- captar novos estudantes para a área.

## 3. Palestras

As palestras serão divididas em dois blocos de duas horas cada. Estão previstos 8 palestrantes, tendo 4 palestrantes em cada bloco. Abaixo, o planejamento inicial das palestras, contendo nome e instituição do palestrante, título da palestra e resumo.

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Bloco 1:

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**Palestra 1** : Renata Raposo Del Vecchio, UFF.
**Título** : Integral Uniform Hypergraphs.
**Resumo** : Although the study of hypergraphs and their structural properties can be considered a fruitful area, with many published articles, the Spectral Theory for hypergraphs is still at an early stage. The search for integral graphs i.e., graphs whose eigenvalues are all integers is one of the relevant problems of Spectral Graph Theory, concerning the various matrices associated with graphs. For hypergraphs however, this topic is still undeveloped. In this work, we present several classes of integral uniform hypergraphs. In particular, we address the problem of characterizing integral hypercycles.

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**Palestra 2** : Gabriel de Morais Coutinho, UFMG.
**Título**: The largest $k$-colourable subgraph and eigenvalues.
**Resumo** : You have $k$ colours and you must put different colours to adjacent vertices. What is the maximum number of vertices you can colour? This is, of course, a hard problem, so we approximate. In this talk, I will discuss how to use semidefinite programming to obtain eigenvalue bounds for this graph parameter.

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**Palestra 3** : Leonardo de Lima, UFPR.
**Título** : On weakly Hadamard diagonalizable graphs.
**Resumo** : A matrix $W \in \mathbb{R}^{n\times n}$ is called weak Hadamard if $W$ has all entries from the set $\{-1, 0, 1\}$ and $W^TW$ is tridiagonal. A graph $G$ is weakly Hadamard diagonalizable (or WHD for short) if the Laplacian matrix of $G$, denoted by $L(G)$, is diagonalizable by a weak Hadamard matrix $W$, i.e, $L(G) = W\Lambda W^{-1}$, where $\Lambda$ is the diagonal matrix of the same order as the graph, and its entries are the eigenvalues of $L(G)$. Consider the following question:

"Let $G$ be a graph on $n$ vertices. Which graphs $G$ are weakly Hadamard diagonalizable?"

It is not difficult to see that a complete graph on $n$ vertices is WHD. In this talk, we address the question above by presenting known results from the recent literature and some new advances.

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**Palestra 4** : Vilmar Trevisan, UFRGS.
**Título** : Eigenvalue Location and applications.
**Resumo** : We address the problem of estimating graph eigenvalues in terms of eigenvalue location, by which we mean determining the number of eigenvalues of a symmetric matrix that lie in any given real interval. Our focus is on two simple linear-time algorithm that works for symmetric matrices whose underlying graph is a tree or a cograph. The algorithms have applications that go beyond estimating eigenvalues of a particular graph, and allow us to obtain properties of an entire class. We illustrate this with applications to the solution of relevant problems in Spectral Graph Theory.

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Bloco 2:

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**Palestra 1** : Celso Marques da Silva Júnior, CEFET/RJ.
**Título** : Ordering trees by $\alpha$-index
**Resumo**: Let $G$ be a graph with adjacency matrix $A(G)$ and diagonal matrix of degrees $Deg(G)$. For every real number $\alpha \in [0,1]$, let $A_\alpha(G) = \alpha Deg(G) + (1-\alpha)A(G)$. In this work, we consider the problem of ordering trees according to the spectral radius of the $A_\alpha$-matrices, the $\alpha$-index, determining those of order $n$ that attain from the third to sixth largest values for this parameter. In particular, these results generalize the already known ordering when the spectral radius of the adjacency or Laplacian matrices is considered, instead of the $\alpha$-index.

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**Palestra 2** : Lucas Siviero Sibemberg, UFRGS.
**Título** : On the Minimum Number of Distinct Eigenvalues: Identifying Defective Seeds.
**Resumo** : The underlying graph $G$ of a symmetric matrix $M = (m_{ij}) \in \mathbb{R}^{n\times n}$ is the graph with vertex set $\{v_1, \ldots, v_n\}$ where a pair $\{v_i, v_j\}$ with $i \neq j$ is an edge if and only if $m_{ij} \neq 0$. For a given graph $G$, let $q(G)$ denote the minimum number of distinct eigenvalues of a symmetric matrix whose underlying graph is $G$. It is well known that $q(T) \geq d(T) + 1$, where $d(T)$ is the diameter of $T$, and a tree $T$ is said to be diminimal if $q(T) = d(T) + 1$. Johnson and Saiago [Johnson, C.R., and Saiago, C.M., *Diameter Minimal Trees*, *Linear and Multilinear Algebra* **64(3)** (2015), 557–571.] introduced an operation on trees called unfolding. For each diameter $d$, there exists a finite set of trees, known as seeds, such that every tree with diameter $d$ is an unfolding of exactly one seed. A seed $S$ is called defective if there is an unfolding of $S$ which is not diminimal. In this talk, we aim to provide a complete characterization of defective seeds with diameter 6 and study how the number of defective seeds grows as the diameter increases.

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**Palestra 3** : Henrique Assumpção, UFMG.
**Título** : Combinatorial parameters of graphs in association schemes.
**Resumo** : Association schemes are combinatorial objects that were first introduced in the 1950's to study statistical experiments, but have since played a central role in spectral and algebraic graph theory in the study of distance-regular graphs and permutation groups. We will discuss how to use the combinatorial and algebraic properties of

schemes and their associated Bose-Mesner algebras to obtain tight bounds for NP-hard graph parameters, such as the clique and coclique numbers. We will also show how to use certain semidefinite programs to obtain eigenvalue bounds for the MAXCUT and MAX 2-SAT parameters of such graphs.

(This is joint work with Gabriel Coutinho)

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**Palestra 4** : Luiz Emílio Allem, UFRGS.

**Título** : On the Minimum Number of Distinct Eigenvalues of Threshold and Chain Graphs.

**Resumo** : Any symmetric matrix $M = (m_{ij}) \in \mathbb{F}^{n\times n}$, where $\mathbb{F}$ is a field, can be associated with a simple graph $G$ with vertex set $[n] = \{1, \ldots, n\}$, where distinct vertices $i$ and $j$ are adjacent if and only if $m_{ij} \neq 0$. The graph $G$ is referred to as the *underlying graph* of $M$. Let $\mathcal{S}(G)$ denote the set of real symmetric matrices whose underlying graph is $G$. We investigate the quantity

$$q(G) = \min\{|\mathrm{DSpec}(A)| : A \in \mathcal{S}(G)\},$$

where $\mathrm{DSpec}(A)$ represents the set of distinct eigenvalues of $A$. In this talk, we present results concerning the minimum number of distinct eigenvalues for threshold and chain graphs. Specifically, we show that for a threshold graph $G$, $q(G) \leq 4$, and for a chain graph $G$, $q(G) \leq 5$.

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